1 Optimization models

1.1 Introduction

A **mathematical static optimization model** consists of an *objective function* and a set of *constraints* expressed in the form of a system of equations or inequalities. Optimization models are used extensively in almost all areas of *decision-making* such as engineering design, and financial portfolio selection.

Optimization Models are made up of three basic ingredients:

- decision variables,
- objective function,
- constraints.

1.2 Examples

Example 1
A factory manufactures two products: P_1 and P_2, using three kinds of resources: R_1, R_2 and R_3. Product P_1 requires 12 units of R_1 and 8 units of R_2. Product P_2 consumes 6 units of R_1, 12 units of R_2 and 10 units of R_3. There is 630 units of R_1 available, 620 of R_2 and 350 of R_3. A factory earns 20$ selling P_1 and 60$ selling P_2. The goal is to maximize total profit by making decision about the amount of P_1 and P_2 to be produced.

Example 2 *(Knapsack problem)*
The task is to load up a container of capacity $V[m^3]$ and $P[kg]$ with objects of N kinds. Each kind of object is characterized by some volume $v_n[m^3]$, weight $p_n[kg]$ and value $c_n[\$]$ $(n = 1, 2, \ldots, N)$. How to maximize the total value of load?

Example 3 *(Assignment problem)*
We have a group of N "applicants" applying for N "jobs,". The cost of assigning the i-th applicant to j-th job is c_{ij}. The objective is to make one-to-one assignment that minimizes total cost.

Example 4
A shipper having M warehouses with supply α_m of goods at his m-th warehouse must ship goods to N geographically dispersed retail centers, each with a given customer demand β_n, which must be met. The unit cost of transportation between the m-th warehouse and the n-th retail center is c_{mn}. The objective is to determine the minimum possible transportation costs.

Example 5 *(The shortest path problem)*
The problem is to determine the best way to traverse a network to get from an origin to a given destination as cheaply as possible. Suppose that in a given network there are M nodes and N arcs (i.e. edges) and a cost c_{ij} associated with each arc (i to j) in the network. Formally, the Shortest Path (SP) problem is to find the shortest (least cost) path from the start node 1 to the finish node M. The cost of the path is the sum of the costs on the arcs in the path.

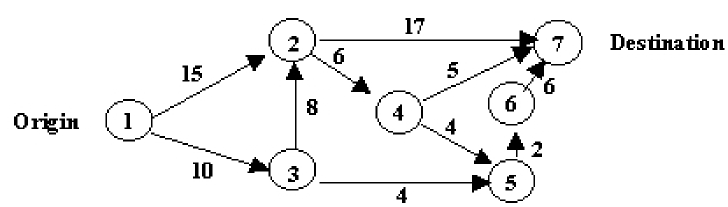

Figure 1: Example of a network