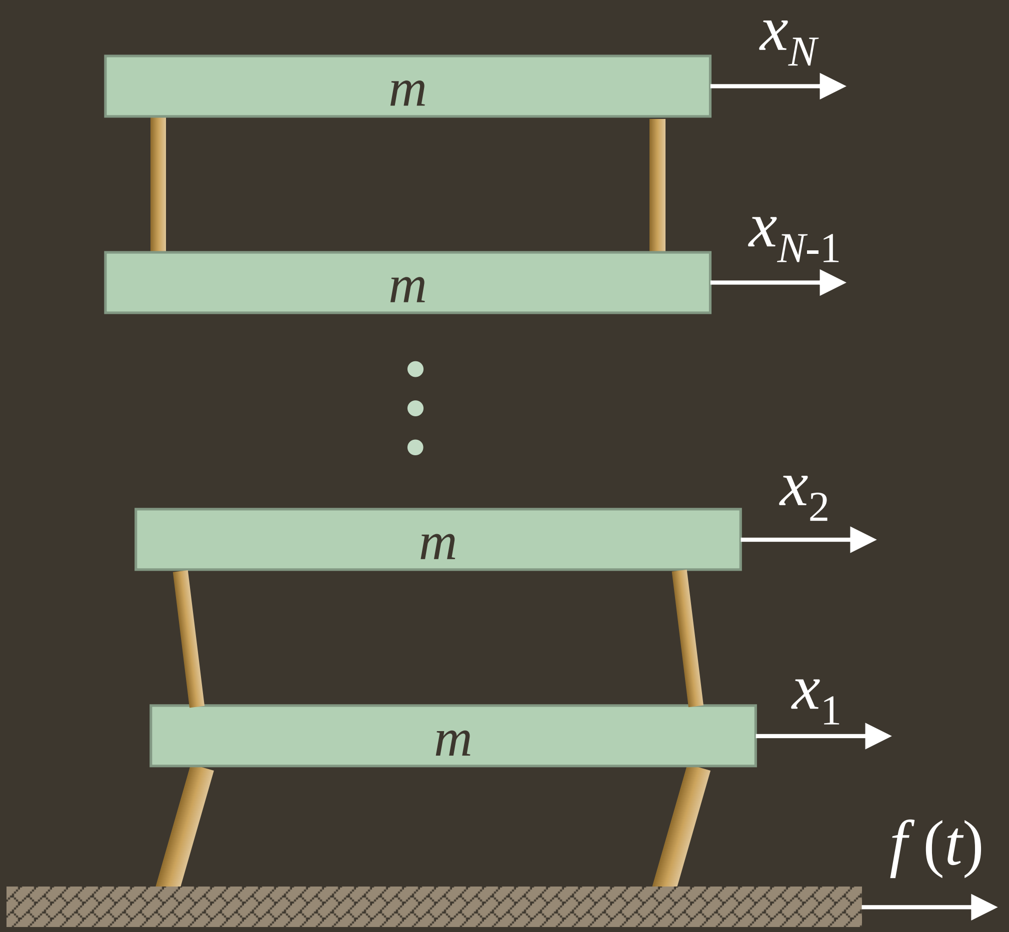
Trzęsienie ziemi i wielopiętrowe budynki

Wstęp do systemów opisywanych równaniami różniczkowymi cząstkowymi

Model budynku wielopiętrowego



$$ma = F_k$$

$$mx''_N = -k\left(x_N - x_{N-1}\right)$$

$$mx''_{N-1} = -k\left(x_{N-1} - x_{N-2}\right) - k\left(x_{N-1} - x_N\right)$$

$$\vdots$$

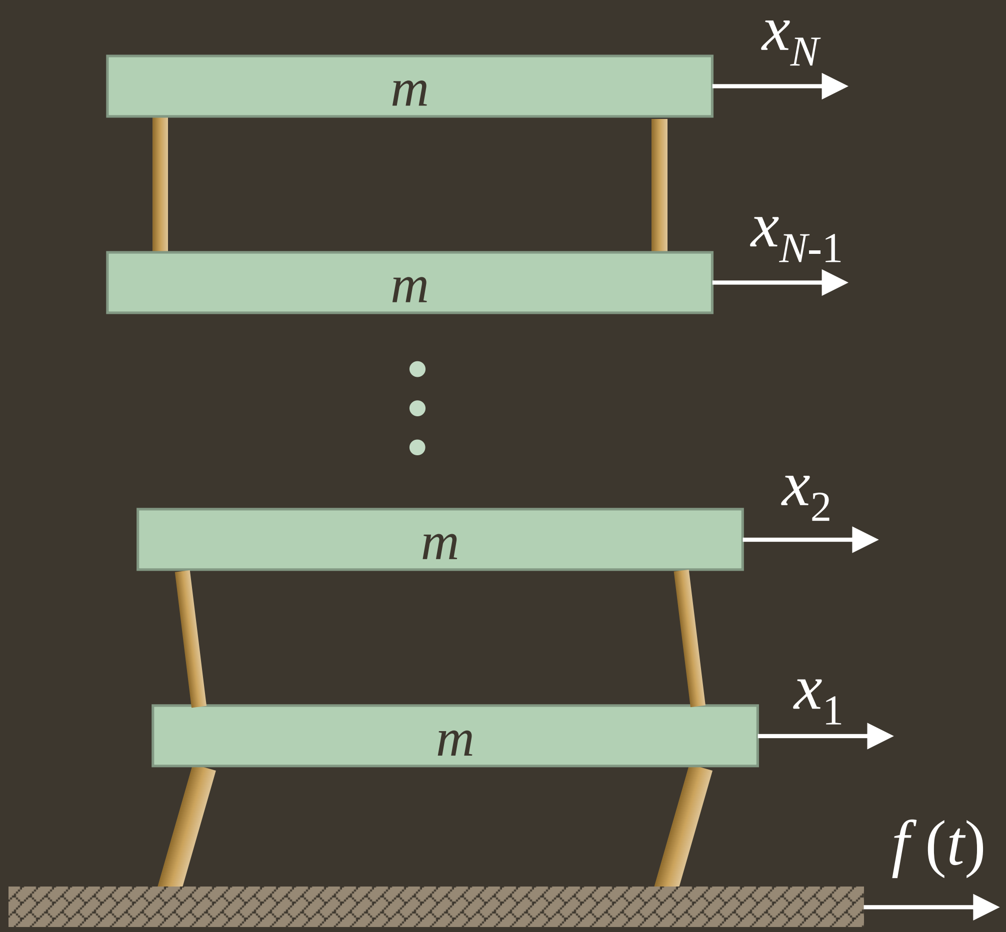
$$mx''_n = -k\left(x_n - x_{n-1}\right) - k\left(x_n - x_{n+1}\right)$$

$$\vdots$$

$$mx''_2 = -k\left(x_2 - x_1\right) - k\left(x_2 - x_3\right)$$

$$mx''_1 = -k\left(x_1 - f(t)\right) - k\left(x_1 - x_2\right)$$

Model budynku wielopiętrowego



$$x_N'' = -(k/m)(x_N - x_{N-1})$$
$$x_{N-1}'' = -(k/m)(-x_N + 2x_{N-1} - x_{N-2})$$
$$\vdots$$
$$x_n'' = -(k/m)(-x_{n+1} + 2x_n - x_{n-1})$$
$$\vdots$$
$$x_2'' = -(k/m)(-x_3 + 2x_2 - x_1)$$
$$x_1'' = -(k/m)(-x_2 + 2x_1 - f(t))$$

Model oscylatora

$$mx''(t) = -kx(t)$$

$$\sin(\omega t - \varphi) = \cos\varphi \sin\omega t - \sin\varphi \cos\omega t$$

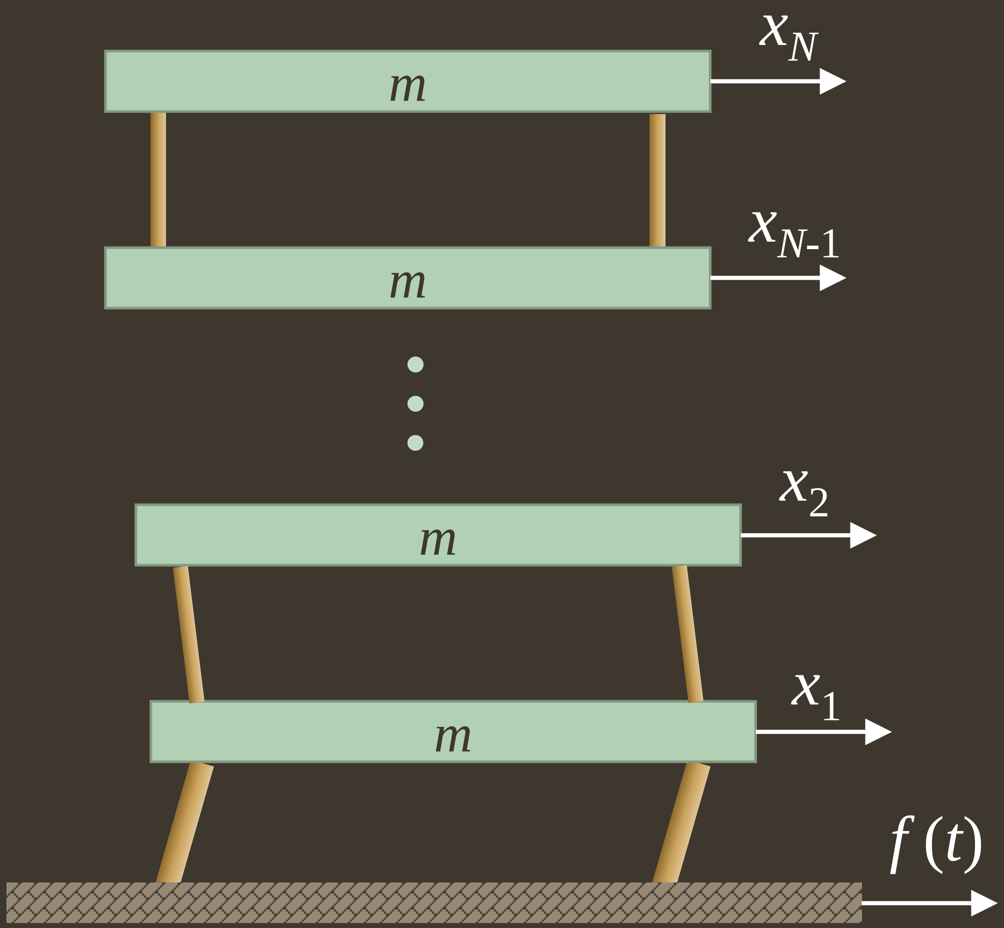
$$\sin(\omega t - \varphi) = A\sin\omega t + B\cos\omega t$$

$$x(t) = C_1 \sin\omega t + C_2 \cos\omega t$$

$$x''(t) = -\omega_0^2 x(t) \longrightarrow \omega_0^2 = \frac{k}{m}$$

Model budynku wielopiętrowego

$$\omega_0^2 = \frac{k}{m}$$



$$x_N'' = -\omega_0^2\left(x_N - x_{N-1}\right)$$

$$x_{N-1}'' = -\omega_0^2\left(-x_N + 2x_{N-1} - x_{N-2}\right)$$

$$\vdots$$

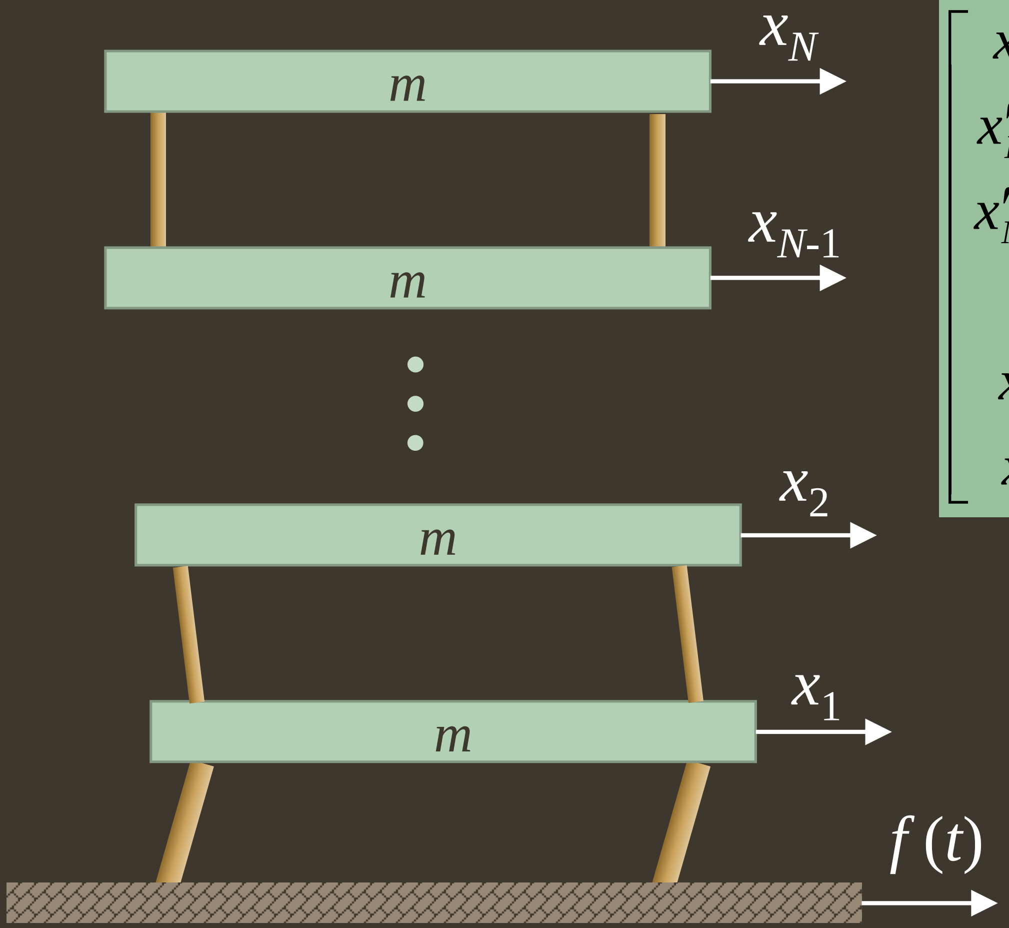
$$x_n'' = -\omega_0^2\left(-x_{n+1} + 2x_n - x_{n-1}\right)$$

$$\vdots$$

$$x_2'' = -\omega_0^2\left(-x_3 + 2x_2 - x_1\right)$$

$$x_1'' = -\omega_0^2\left(-x_2 + 2x_1\right) + \omega_0^2 f(t)$$

Model budynku wielopiętrowego



$$\begin{bmatrix} x_N'' \\ x_{N-1}'' \\ x_{N-2}'' \\ \vdots \\ x_2'' \\ x_1'' \end{bmatrix} = -\omega_0^2 \begin{bmatrix} 1 & -1 & 0 & 0 & \cdots & 0 \\ -1 & 2 & -1 & 0 & \cdots & 0 \\ 0 & -1 & 2 & -1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & -1 & 2 & -1 \\ 0 & \cdots & \cdots & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_N \\ x_{N-1} \\ x_{N-2} \\ \vdots \\ x_2 \\ x_1 \end{bmatrix} + \omega_0^2 f(t) \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{x}'' = -\omega_0^2 \mathbf{K}\mathbf{x} + \omega_0^2 f(t)\mathbf{z}$$

$$\mathbf{x}'' + \omega_0^2 \mathbf{K}\mathbf{x} = \omega_0^2 f(t)\mathbf{z}$$

Analiza modelu budynku wielopiętrowego

$$\mathbf{x}'' + \omega_0^2 \mathbf{K}\mathbf{x} = \omega_0^2 f(t)\mathbf{z}$$

$$\mathbf{x}^T \mathbf{K}\, \mathbf{x} > 0, \quad \mathbf{x} \neq 0$$

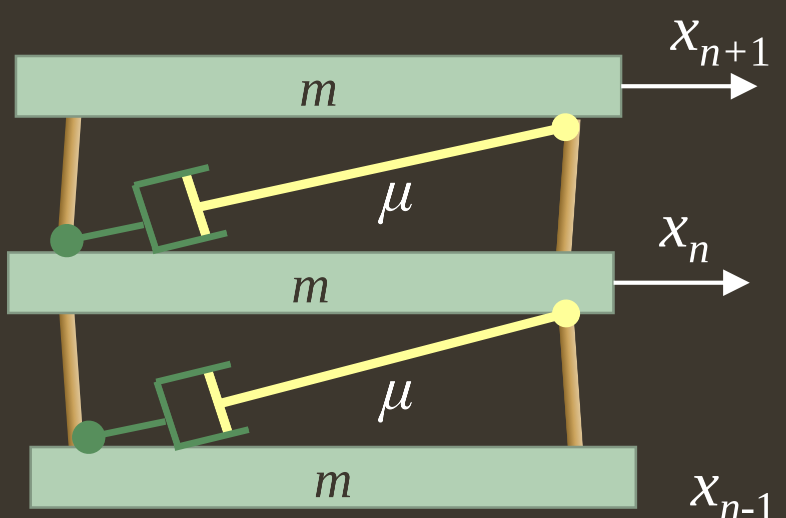
$$\mathbf{K} = \begin{bmatrix} 1 & -1 & 0 & 0 & \cdots & 0 \\ -1 & 2 & -1 & 0 & \cdots & 0 \\ 0 & -1 & 2 & -1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & -1 & 2 & -1 \\ 0 & \cdots & \cdots & 0 & -1 & 2 \end{bmatrix}$$

Macierz **K** jest rzeczywista, symetryczna, dodatnio określona, ma zatem rzeczywiste dodatnie wartości własne

$$\mathbf{x} = a_1(t)\mathbf{u}_1 + \ldots + a_N(t)\mathbf{u}_N$$

$\mathbf{u}_n$ – wektory własne macierzy **K**

Model z tłumieniem



$$ma = F_k + F_\mu$$

$$F_\mu^{(N)} = -\mu\left(x'_N - x'_{N-1}\right)$$

$$F_\mu^{(N-1)} = -\mu\left(x'_{N-1} - x'_{N-2}\right) - \mu\left(x'_{N-1} - x'_N\right)$$

$$\vdots$$

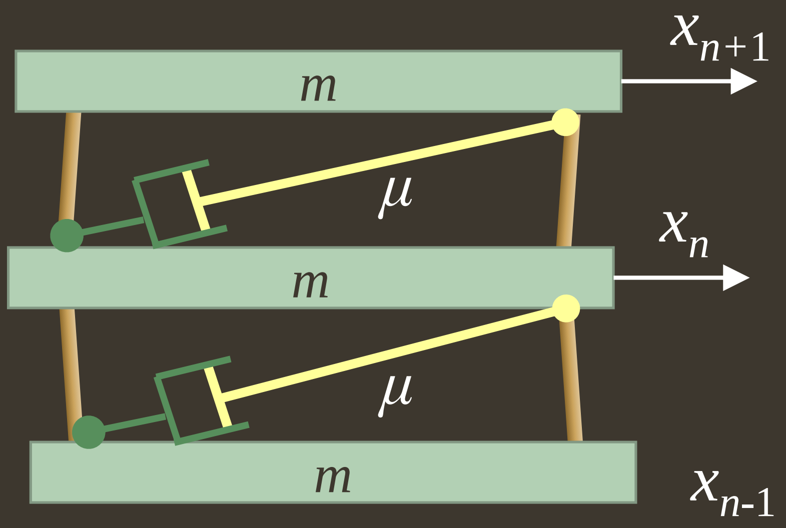
$$F_\mu^{(n)} = -\mu\left(x'_n - x'_{n-1}\right) - \mu\left(x'_n - x'_{n+1}\right)$$

$$\vdots$$

$$F_\mu^{(2)} = -\mu\left(x'_2 - x'_1\right) - \mu\left(x'_2 - x'_3\right)$$

$$F_\mu^{(1)} = -\mu\left(x'_1 - f'(t)\right) - \mu\left(x'_1 - x'_2\right)$$

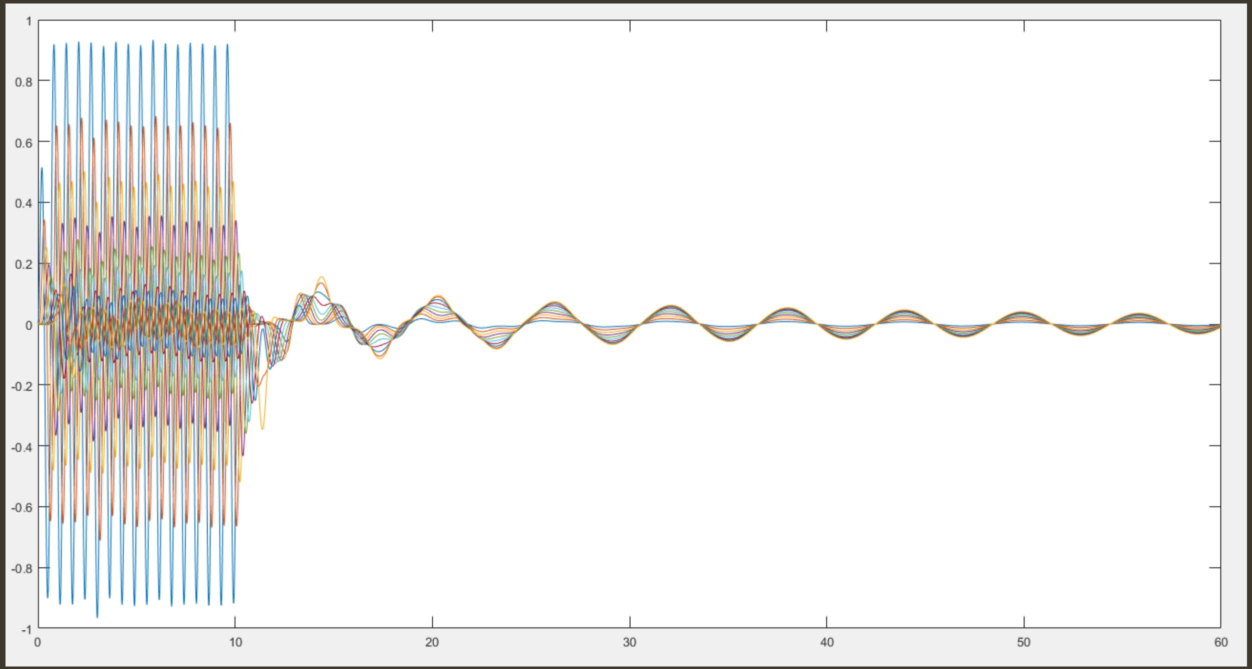
Model z tłumieniem

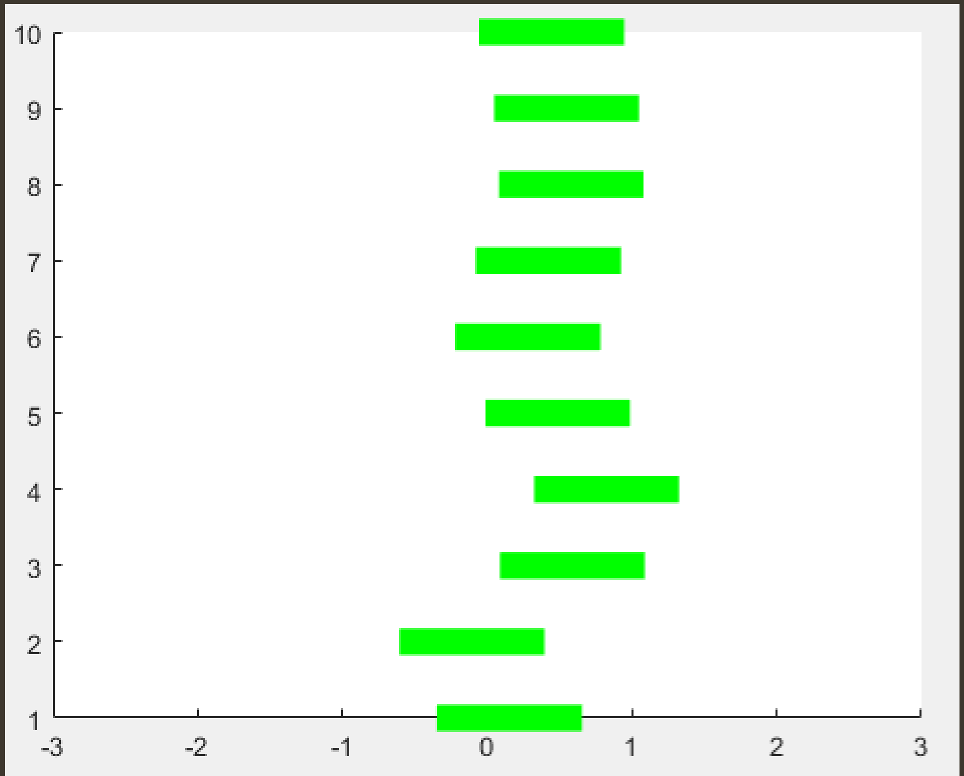


$$\delta = \frac{\mu}{m}$$

$$\mathbf{x}'' + \delta\,\mathbf{K}\mathbf{x}' + \omega_0^2\mathbf{K}\mathbf{x} = \left(\omega_0^2 f(t) + \delta\, f'(t)\right)\mathbf{z}$$

Symulacja komputerowa – przykład z animacją





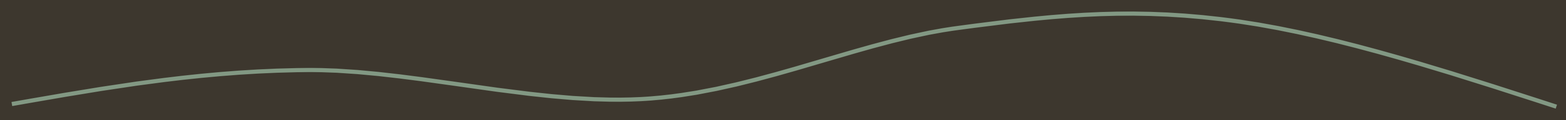
Symulacja komputerowa – kod

```
function dX = rrStropy(t,X)

    k = 10; m = 0.2;
    mu = 2;  % tlumienie
    N = length(X)/2; % liczba stropów

    %wejscie
    A = 1;      % amplituda u
    om = 10;     % czestotliwosc u
    if t < 10
        u = A*cos(om*t);
        uprim = A*om*cos(om*t);
    else
        u = 0;
        uprim = 0;
    end

    x = X(1:N);
    y = X(N+1:2*N);     % pierwsza polowa X to x a druga polowa X to y
    dx = y;
    dy = zeros(N,1);
    dy(1) = (-k/m)*(-x(2)+2*x(1)-u) - mu*(-y(2)+2*y(1)-uprim);   % pierwszy strop
    for n = 2:N-1
        dy(n) = (-k/m)*(-x(n+1)+2*x(n)-x(n-1)) - mu*(-y(n+1)+2*y(n)-y(n-1));
    end
    dy(N) = (-k/m)*(x(N)-x(N-1)) - mu*(y(N)-y(N-1));   % ostatni strop

    dX = [dx; dy];

end
```

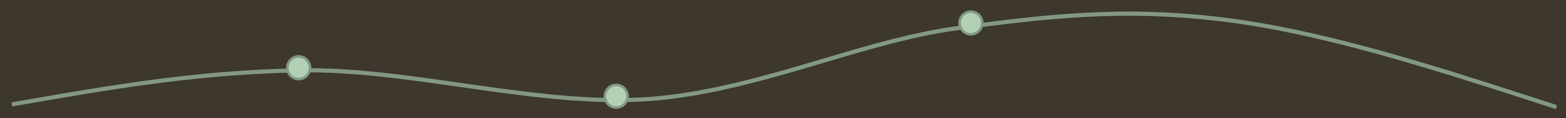
Symulacja komputerowa – kod

```
clear
N = 10; % liczba stropów
X0 = zeros(2*N,1);
tspan = [0 60];

options = odeset('MaxStep',0.1);
[T,X] = ode45(@rrStropy, tspan, X0, options);
X = X(:,1:N);

figure(1)
plot(T,X)
figure(2)
pause
for t = 1:length(T)
    for n = 1:N
        hold on
        plot([X(t,n) X(t,n)+1],[n n],'g','LineWidth',10)
        xlim([-3 3])
        hold off
    end
    drawnow
    cla
end
```

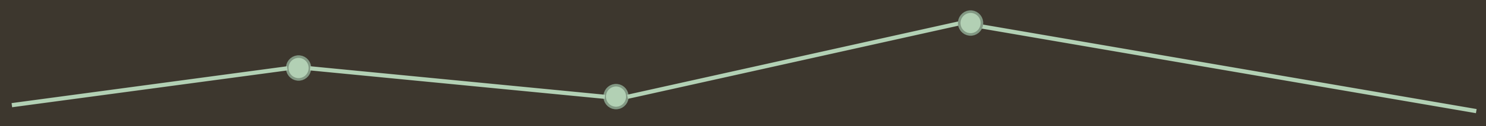
Równanie falowe 1D



Równanie falowe 1D

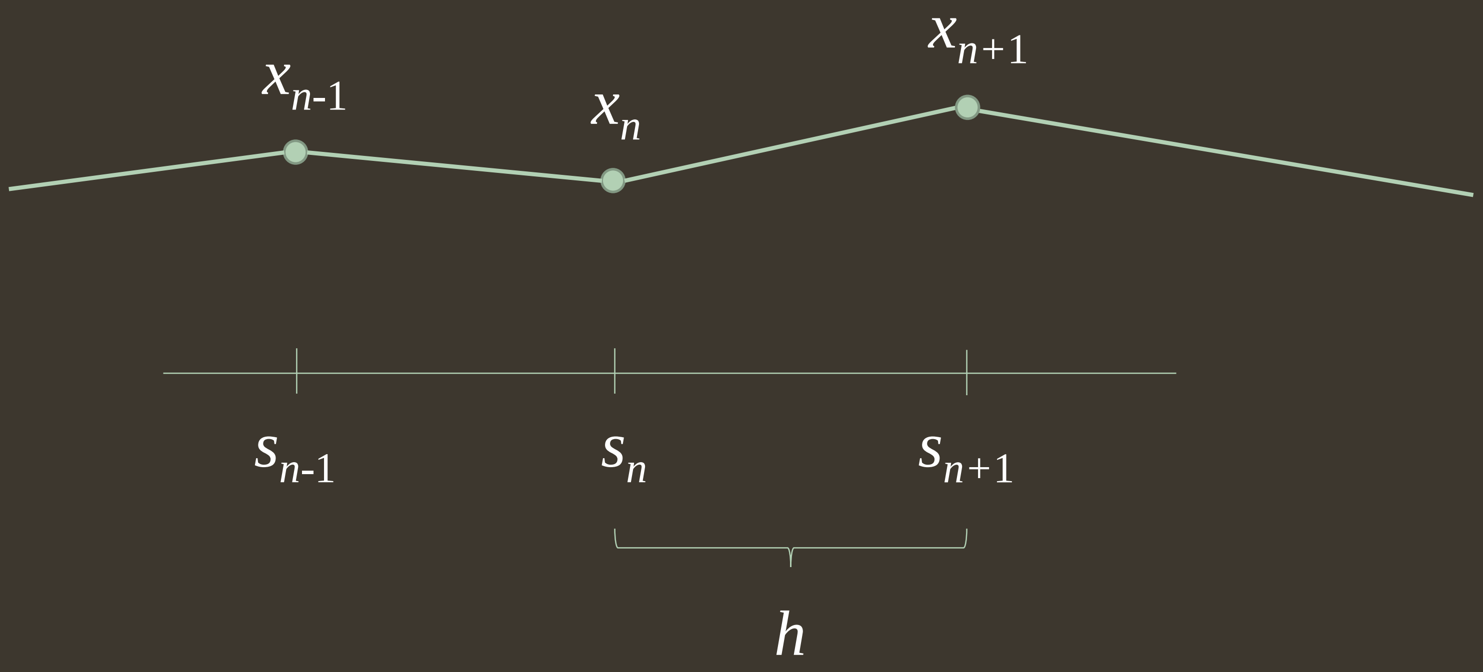


Równanie falowe 1D



Równanie falowe 1D

L – długość struny
ρ – gęstość struny



$x = u(s,t)$

$s_n = nh = n\mathrm{L/N}$

$x_n(t) = u(s_n,t)$

$$\rho h x_n''(t) = -\left(k/h\right)\left[\left(x_n(t) - x_{n+1}(t)\right) + \left(x_n(t) - x_{n-1}(t)\right)\right]$$

Równanie falowe 1D

$$\rho h x_n''(t) = \frac{k}{h}\left(x_{n+1}(t) - 2x_n(t) + x_{n-1}(t)\right)$$

$$\rho x_n''(t) = \frac{k}{h^2}\left(u(s_n + h, t) - 2u(s_n, t) + u(s_n - h, t)\right)$$

$$\frac{f(s+h) - 2f(s) + f(s-h)}{h^2} \approx f''(s)$$

$$\rho \frac{\partial^2 u}{\partial t^2} = k \frac{\partial^2 u}{\partial s^2}$$

Równanie falowe 1D

$$\rho \frac{\partial^2 u}{\partial t^2} = k \frac{\partial^2 u}{\partial s^2}$$

$$c^2 = \frac{k}{\rho}$$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial s^2}$$

Ważne pojęcia:

– układy równań sprzężonych

– częstotliwość drgań własnych

– wektory własne

– równanie falowe