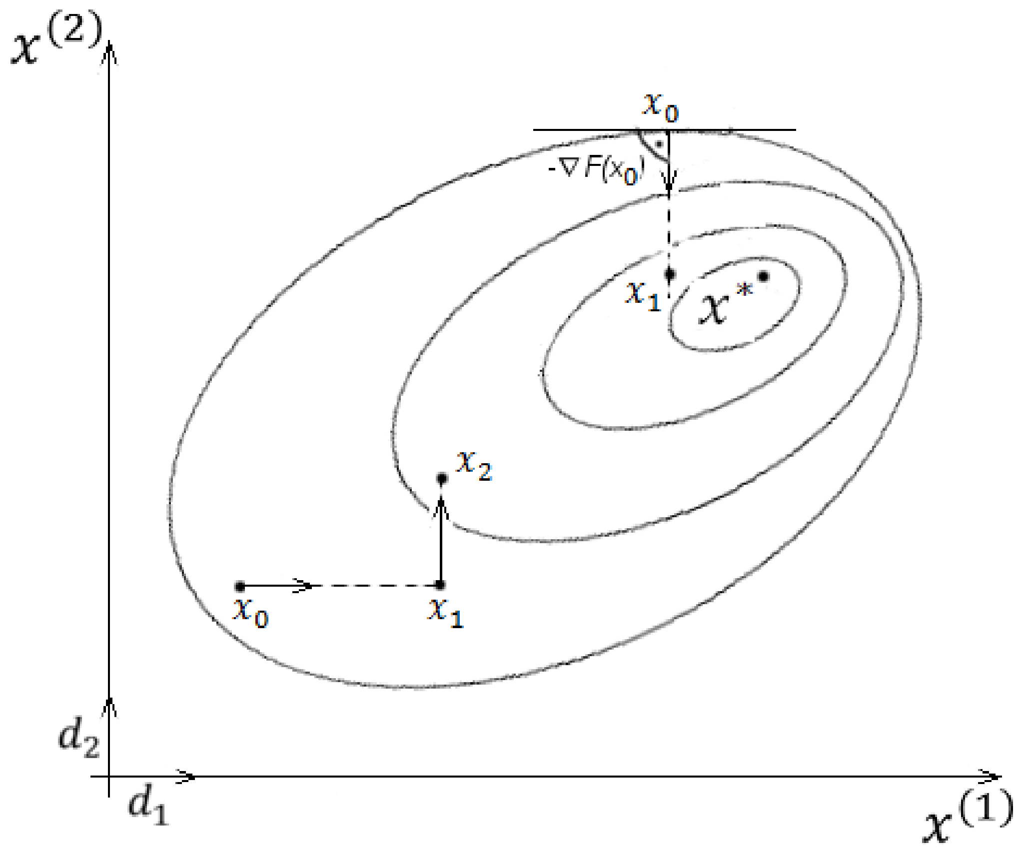
Politechnika Wrocławska

# Gradientowe metody optymalizaci

informacje dodatkowe

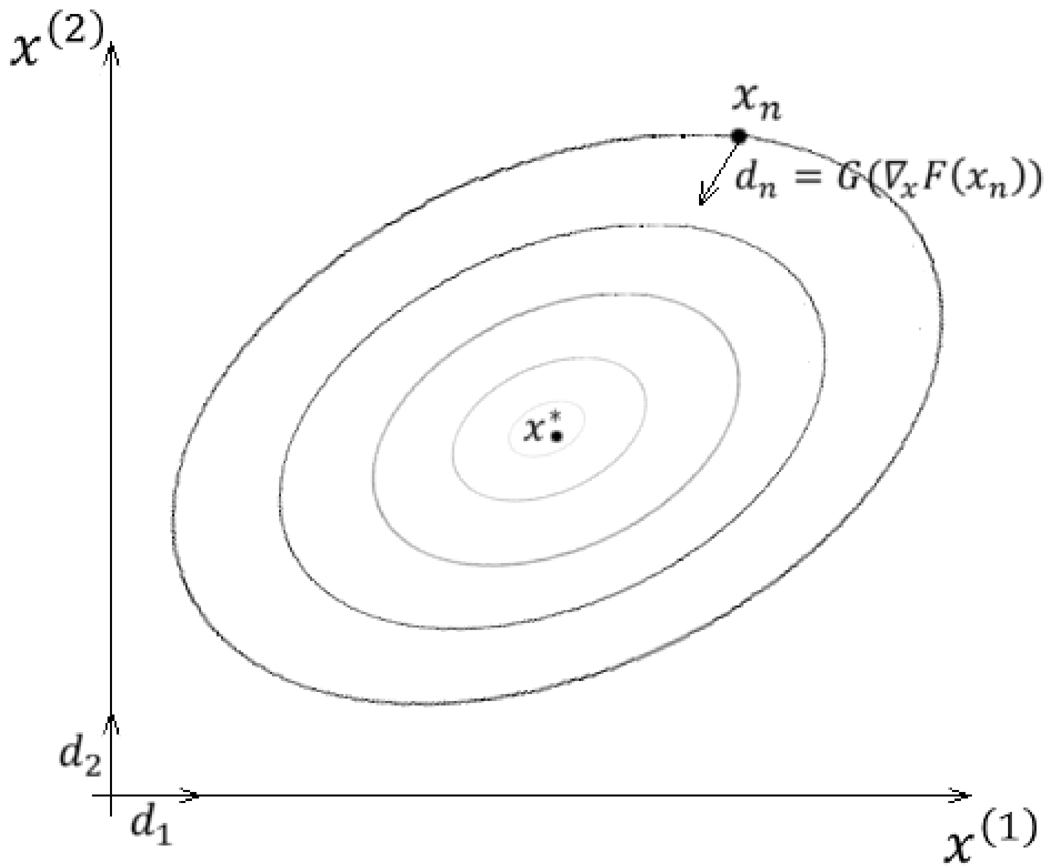
Politechnika Wrocławska

# Wybór kierunku poszukiwań




- Kierunki bazowe i ich modyfikacje – metody bezgradientowe.
- Kierunki oparte na gradiencie funkcji – metody gradientowe.

# Metody gradientowe

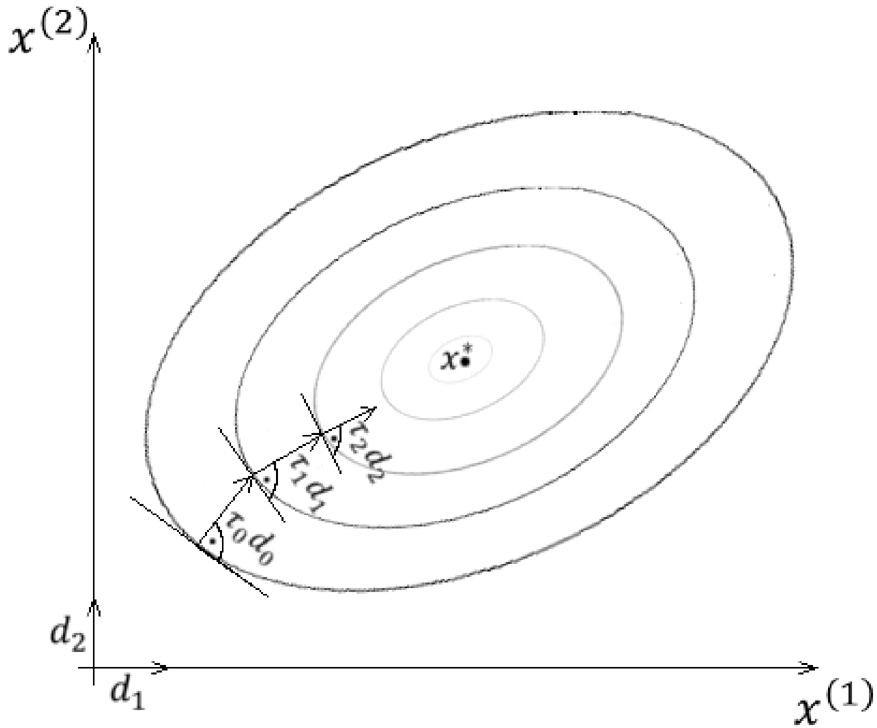



$x_{n+1} = x_n + \tau_n d_n$
$d_n = G(\nabla_x F(x_n))$
$\tau_n$- krok procedury

# Metoda gradientu prostego

$$x_{n+1} = x_n + \tau_n d_n$$

$$d_n = -\nabla_x F(x_n)\,; \tau_n > 0,\ \lim_{n\to\infty} \tau_n = \tau,\ \sum_{n=0}^{\infty} \tau_n = \infty$$




$$\|x_{n+1} - x_n\| = \|\tau_n d_n\| < \varepsilon$$

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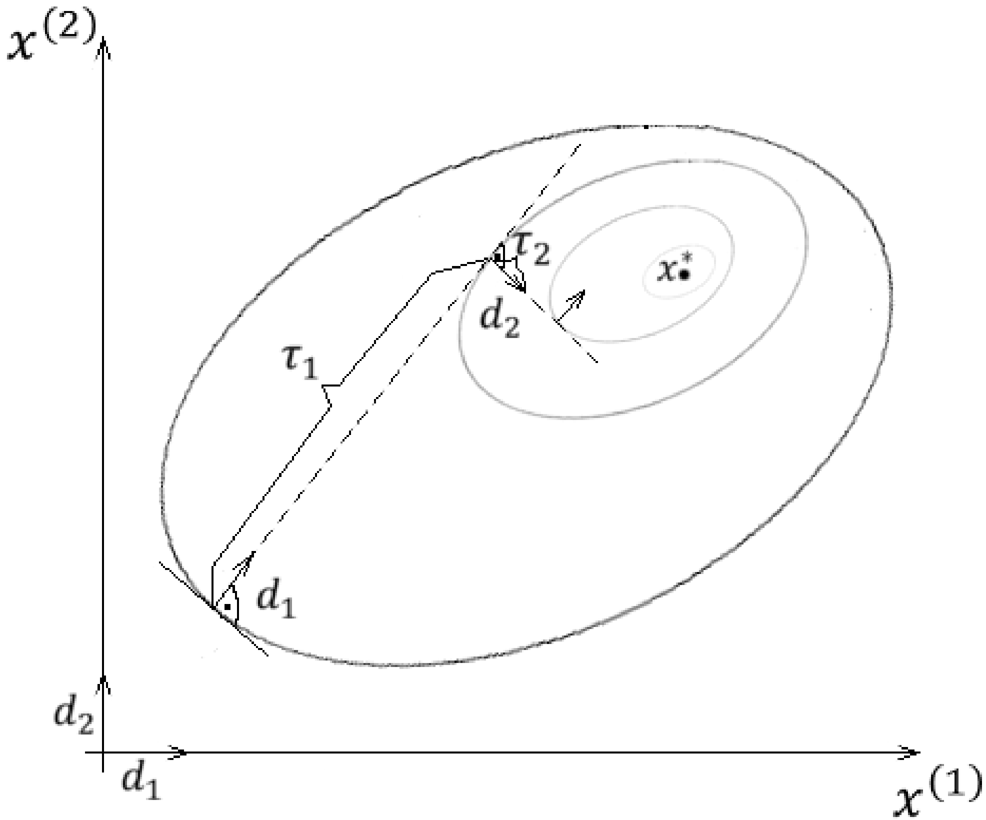
# Metoda najszybszego spadku

$x_{n+1} = x_n + \tau_n d_n$

$d_n = -\nabla_x F(x_n),\ \tau_n$- minimum w kierunku $d_n$




$\|x_{n+1} - x_n\| < \varepsilon$

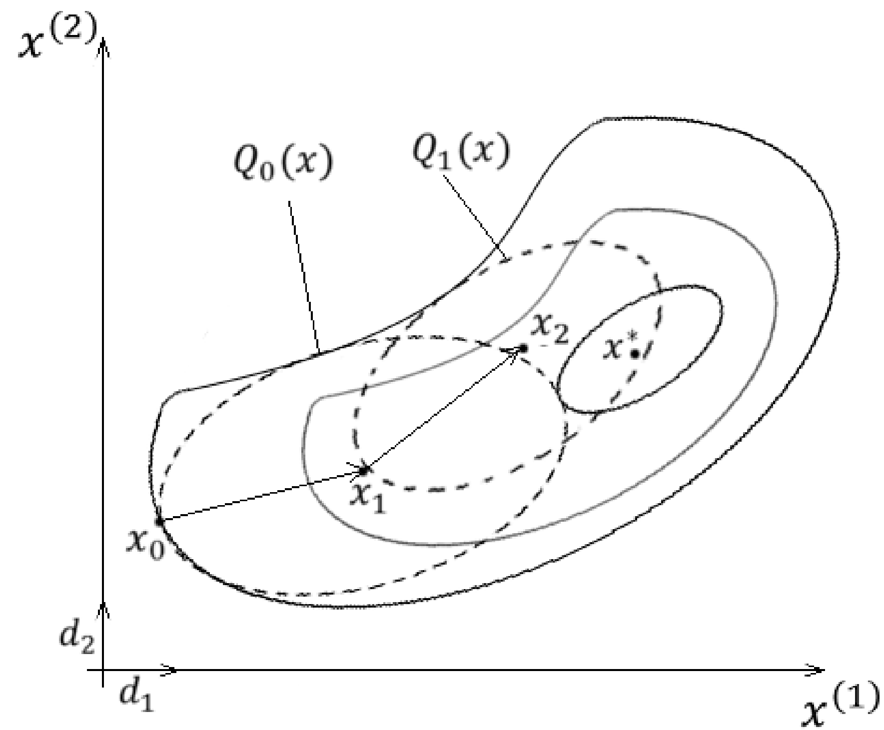
# Metoda Newtona

$$F(x) = \underbrace{F(x_0) + (x - x_0)^T \nabla_x F(x_0) + \frac{1}{2}(x - x_0)^T H(x_0)(x - x_0)}_{Q\quad(x)} + 0_3(\|x - x_0\|)$$

$$\nabla_x Q(x) = \nabla_x F(x_0) + H(x_0)(x^* - x_0) = O_S$$

$$x^* = x_0 - H^{-1}(x_0)\nabla_x F(x_0)$$

$$x_{n+1} = x_n - H^{-1}(x_n)\nabla_x F(x_n)$$



# Metoda Fletchera-Reevesa (gradientu sprzężonego)

Krok 0: $\mathrm{z}_1 = \mathrm{x}_0,\ s = 1,\ d_1 \coloneqq -\nabla_x F(z_1)$

Krok 1: $z_{s+1} \coloneqq z_s + \tau_s d_s$

$\tau_s \rightarrow$ minimum w kierunku $d_s$

Jeśli $\|\tau_s d_s\| < \varepsilon$ (STOP)

w przeciwnym razie idź do kroku 2

Krok 2: $d_{s+1} \coloneqq -\nabla_x F(z_{s+1}) + \frac{\|\nabla_x F(z_{s+1})\|}{\|\nabla_x F(z_s)\|} d_s$

$s \coloneqq s + 1,$ idź do kroku 1

$d_1, d_2, \dots, d_S$ - kierunki sprzężone dla formy kwadratowej

# Metoda zmiennej metryki

Krok 0: $z_1 = x_0$, $d_1 = -D_1 \nabla_x F(z_1)$, $D_1 = I$, $s$=1

Krok 1: $z_{s+1} = z_s + \tau_s d_s$ gdzie $\tau_s$ - minimum w kierunku $d_s$

Jeśli $\|\tau_s d_s\| < \varepsilon$ (STOP)

w przeciwnym razie idź do kroku 2.

Krok 2: $d_{s+1} = -D_{s+1} \nabla_x F(z_{s+1})$

$D_{s+1} = D_s + \frac{p_s p_s^T}{p_s^T q_s} - \frac{D_s q_s q_s^T D_s}{q_s^T D_s q_s}$, gdzie

$p_s = \tau_s d_s$, $q_s = \nabla_x F(z_{s+1}) - \nabla_x F(z_s)$

$s := s+1$ idź do kroku 1.

$D_{S+1} \approx H^{-1}(x_{S+1})$