Laplace transform

$$\mathscr{L}\left[f\left(t\right)\right] \equiv \int_0^\infty f\left(t\right)e^{-st}dt, \quad \text{where } s \text{ is complex variable.}$$

Important properties of Laplace transform:

1. $\mathscr{L}\left[a_1 f_1\left(t\right) + a_2 f_2\left(t\right)\right] = a_1 \mathscr{L}\left[f_1\left(t\right)\right] + a_2 \mathscr{L}\left[f_2\left(t\right)\right]$, where $a_1, a_2 \in \mathscr{R}$.
2. $\mathscr{L}\left[f^{(n)}\left(t\right)\right] = s^n \mathscr{L}\left[f\left(t\right)\right] - s^{n-1} f\left(0\right) - s^{n-2} f'\left(0\right) - \ldots - f^{(n-1)}\left(0\right)$.
3. $\mathscr{L}\left[\int_0^t f\left(u\right)du\right] = \frac{1}{s}\mathscr{L}\left[f\left(t\right)\right]$.

$f(t)$	$\mathscr{L}\left[f(t)\right]$	$f(t)$	$\mathscr{L}\left[f(t)\right]$
$1(t)$	$\frac{1}{s}$	e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$	$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$
$\cos at$	$\frac{s}{s^2+a^2}$	$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$
$t\sin at$	$\frac{2as}{(s^2+a^2)^2}$	$\frac{t^n}{n!}, n \in \mathscr{N}$	$\frac{1}{s^{n+1}}$
$t\cos at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$	$e^{at}\cdot\frac{t^n}{n!}, n \in \mathscr{N}$	$\frac{1}{(s-a)^{n+1}}$

Table 1: Commonly used transformations

The method of least squares:

$$a = \left(\mathbf{U}\mathbf{U}^T\right)^{-1}\mathbf{U}\mathbf{Y}^T$$

The Maximum Likelihood Method:

$$a^* \rightarrow \max_a \prod_{n=1}^N f_z\left(h^{-1}(y_n, z_n), w_n\right)|J|,$$

where:

$$w = h(y, z), \qquad J = \frac{\partial h^{-1}}{\partial w}$$