Computer Science

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Systems Modelling and Analysis

Choose yourself and new technologies

L.17.c Numerical optimization methods – multidimensional search using derivatives

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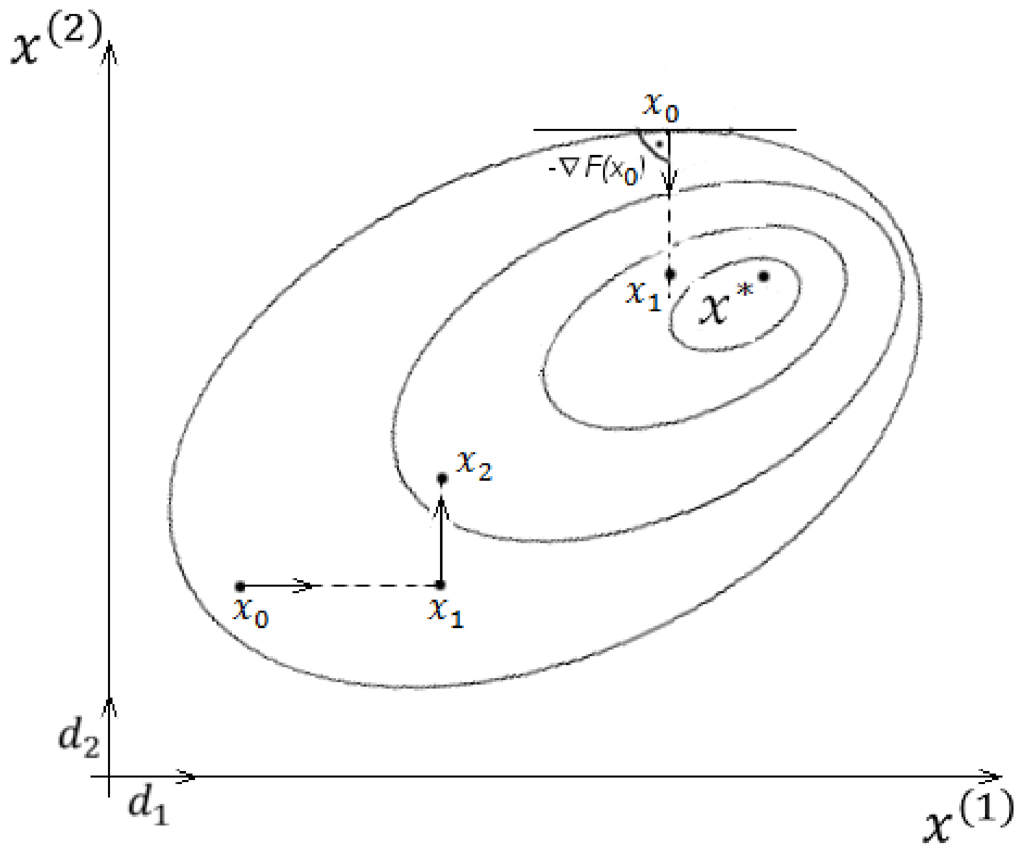
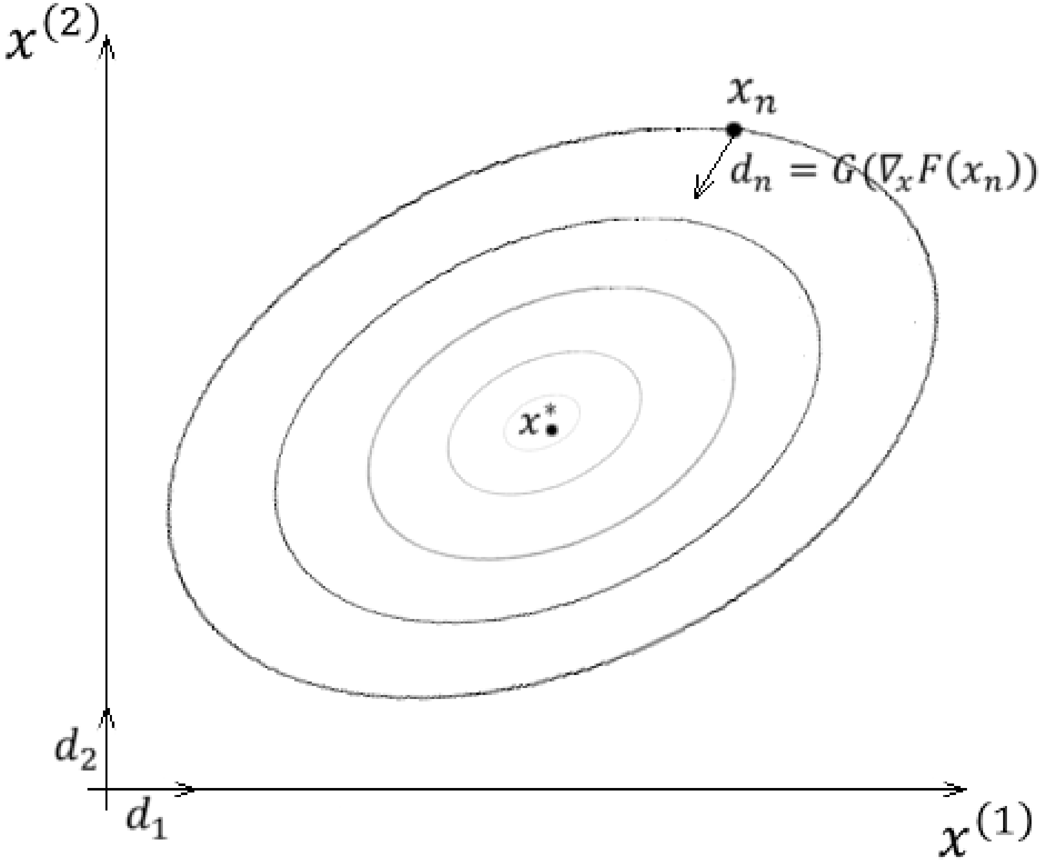
Choice of the search direction



- Basis of search directions – non-gradient methods.
- Search directions based on gradient vectors – gradient-based methods.

Gradient based methods

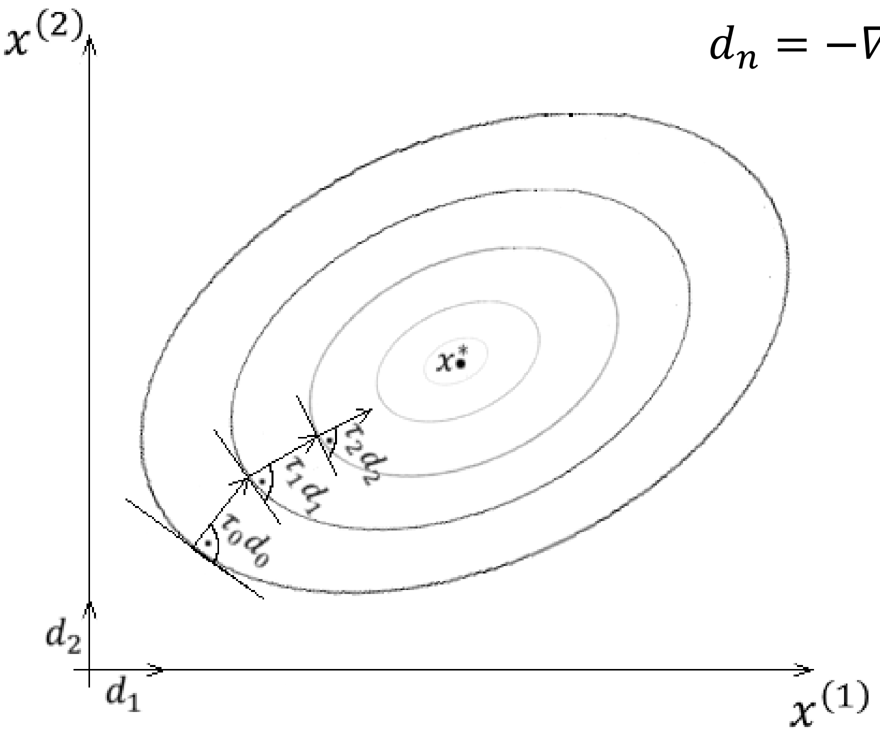


$x_{n+1} = x_n + \tau_n d_n$
$d_n = G(\nabla_x F(x_n))$
τ_n – step size

The gradient descent method

$$x_{n+1} = x_n + \tau_n d_n$$

$$d_n = -\nabla_x F(x_n)\ ; \tau_n > 0,\ \lim_{n\to\infty} \tau_n = \tau,\ \sum_{n=0}^{\infty} \tau_n = \infty$$



$$\|x_{n+1} - x_n\| = \|\tau_n d_n\| < \varepsilon$$

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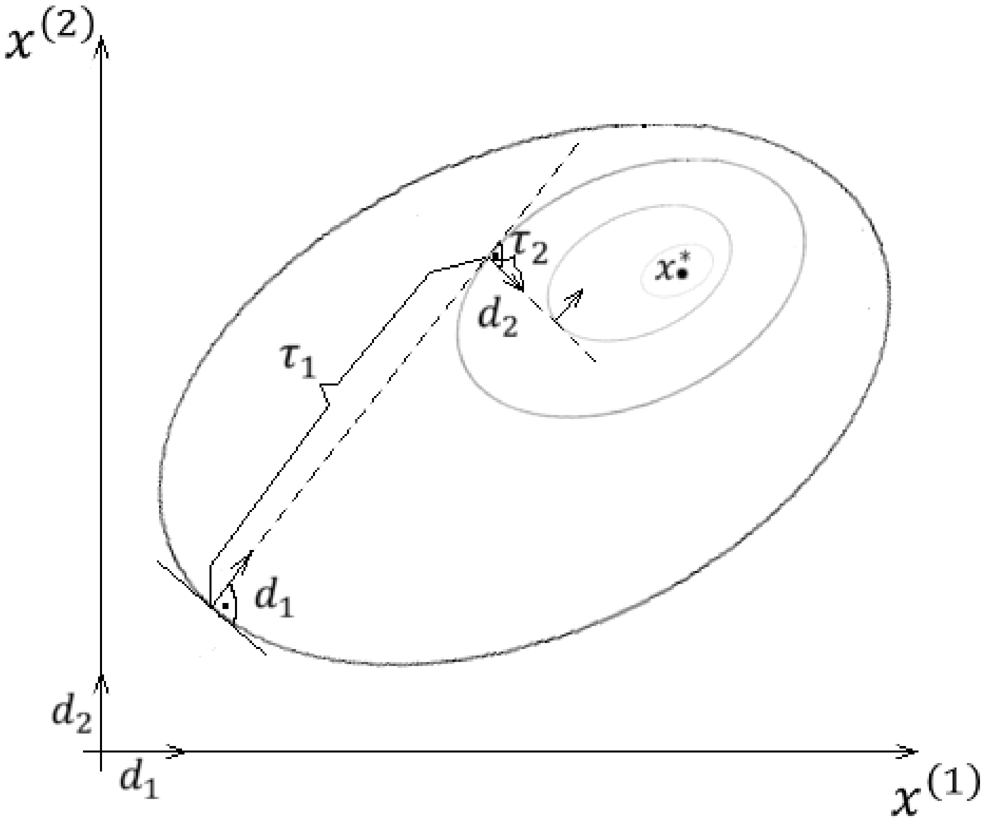
The gradient descent method



$$x_{n+1} = x_n + \tau_n d_n$$

$d_n = -\nabla_x F(x_n)$, τ_n – optimal step size along the direction d_n

$$\|x_{n+1} - x_n\| < \varepsilon$$

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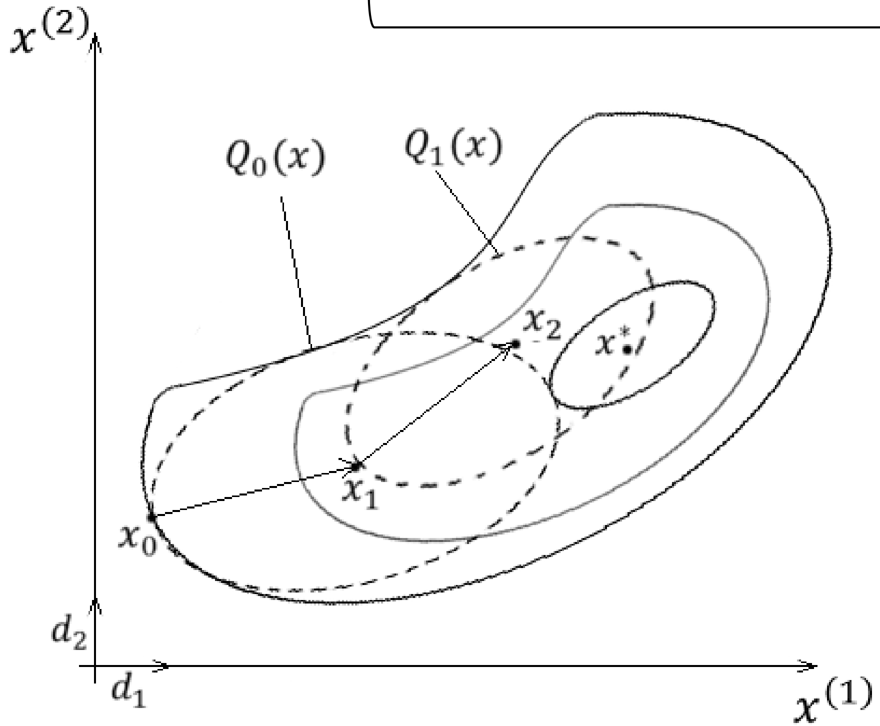
Newton's method

$$F(x) = \underbrace{F(x_0) + (x - x_0)^T \nabla_x F(x_0) + \frac{1}{2}(x - x_0)^T H(x_0)(x - x_0)}_{Q\quad(x)} + 0_3(\|x - x_0\|)$$



$$\nabla_x Q(x) = \nabla_x F(x_0) + H(x_0)(x^* - x_0) = O_S$$

$$x^* = x_0 - H^{-1}(x_0)\nabla_x F(x_0)$$

$$x_{n+1} = x_n - H^{-1}(x_n)\nabla_x F(x_n)$$

Fletcher-Reeves method of conjugate gradients

Step 0: $\mathrm{z}_1 = \mathrm{x}_0,\ s = 1,\ d_1 := -\nabla_x F(z_1)$

Step 1: $z_{s+1} := z_s + \tau_s d_s$

$\tau_s \rightarrow$ optimal step size along the direction d_s

If $\|\tau_s d_s\| < \varepsilon$ (STOP)

otherwise go to 2

Step 2: $d_{s+1} := -\nabla_x F(z_{s+1}) + \frac{\|\nabla_x F(z_{s+1})\|}{\|\nabla_x F(z_s)\|} d_s$

$s := s + 1,$ go to 1

$d_1, d_2, \dots, d_S$ – conjugate directions

Variable metric methods

$z_1 = x_0$

$d_1 = -D_1 \nabla_x F(z_1) \quad D_1 = I$

$z_{s+1} = z_s + \tau_s d_s$ τ_s – optimal step size along the direction d_s

$d_{s+1} = -D_{s+1} \nabla_x F(z_{s+1})$

$$D_{s+1} = D_s + \frac{p_s p_s^T}{p_s^T q_s} - \frac{D_s q_s q_s^T D_s}{q_s^T D_s q_s},$$

$p_s = \tau_s d_s$

$q_s = \nabla_x F(z_{s+1}) - \nabla_x F(z_s)$

$D_{s+1} \approx H^{-1}(x_{s+1})$