

# Computer Science

## Jerzy Świątek

# Systems Modelling and Analysis

*Choose yourself and new technologies*

### L.2a. Determination of the plant parameters



**HUMAN CAPITAL**  
HUMAN – BEST INVESTMENT!



Wrocław University of Technology

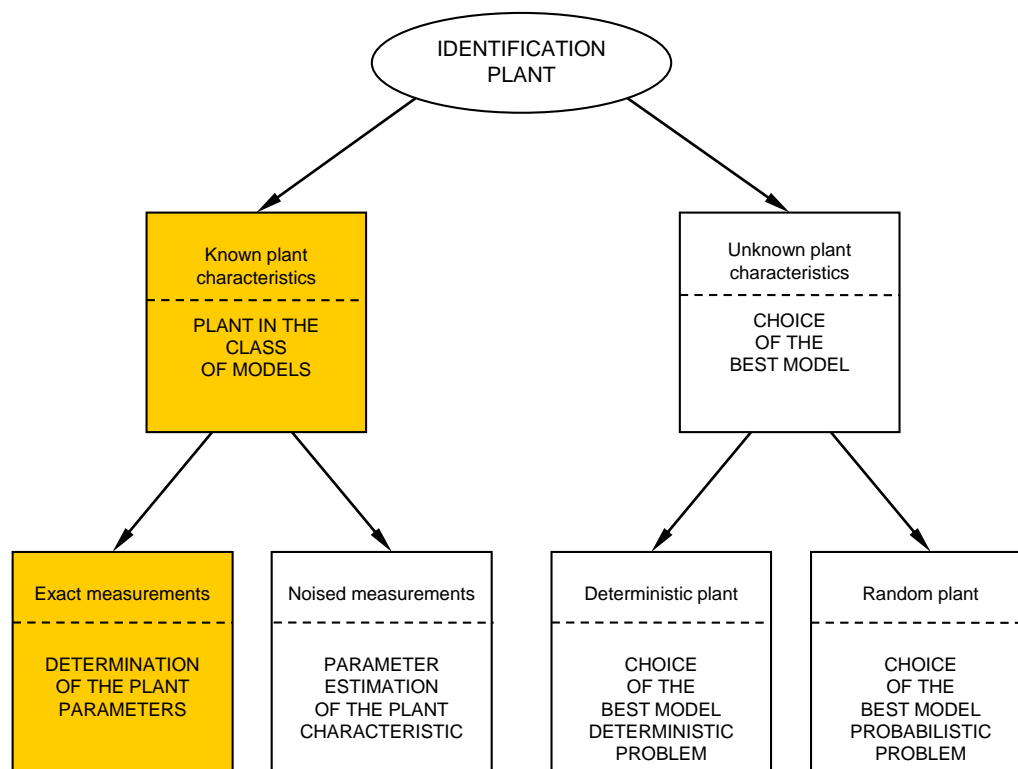
EUROPEAN  
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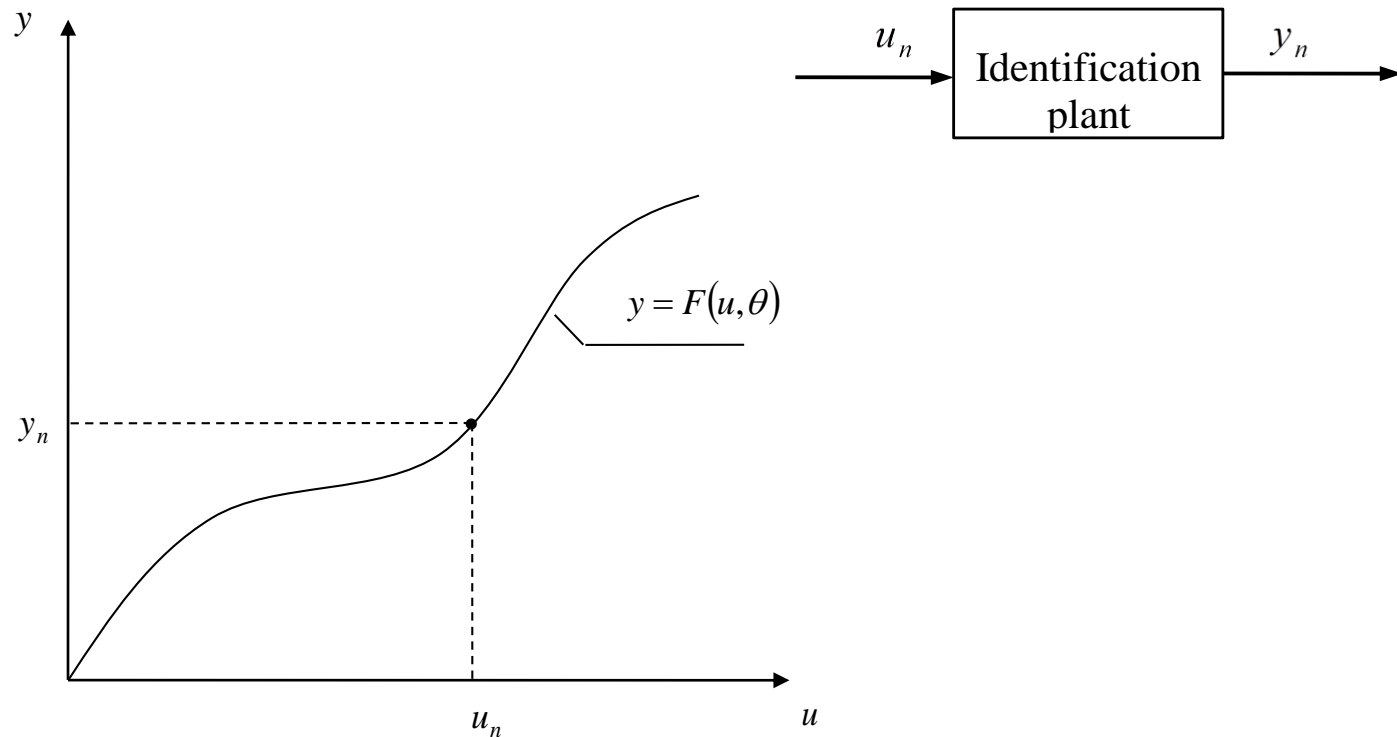
# Typical identification tasks





# Determination of the plant parameters (1)

## Exact Measurements





# Determination of the plant parameters (2)

- Problem formulation

Static plant characteristic:  $y = F(u, \theta)$

$F$  – known function

$u$  – input vector  $u \in \mathcal{U} \subseteq \mathcal{R}^S$   $\mathcal{U}$  – input domain

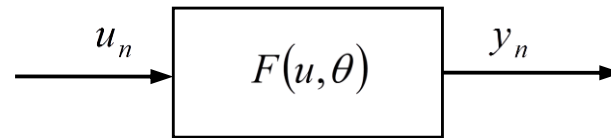
$y$  – output vector  $y \in \mathcal{Y} \subseteq \mathcal{R}^L$   $\mathcal{Y}$  – output domain

$\theta$  – unknown vector of the plant characteristics parameters,  $\theta \in \Theta \subseteq \mathcal{R}^R$

$\Theta$  – parameters domain



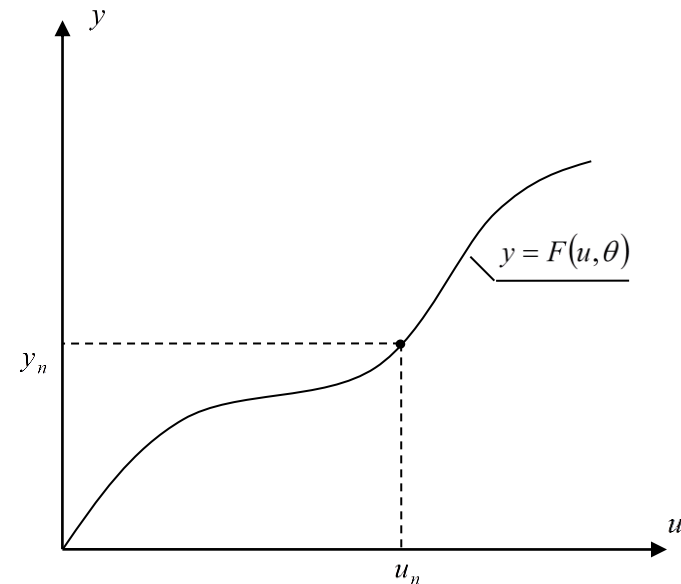
# Determination of the plant parameters (3)



Measurements:

$$U_N = [u_1 \quad u_2 \quad \cdots \quad u_N],$$

$$Y_N = [y_1 \quad y_2 \quad \cdots \quad y_N]$$





# Determination of the plant parameters (4)

System of equations:

$$y_n = F(u_n, \theta), \quad n = 1, 2, \dots, N$$

can be written

$$[y_1 \quad y_2 \quad \dots \quad y_N] = [F(u_1, \theta) \quad F(u_2, \theta) \quad \dots \quad F(u_N, \theta)]$$

For

$$[F(u_1, \theta) \quad F(u_2, \theta) \quad \dots \quad F(u_N, \theta)] \stackrel{df}{=} \bar{F}(U_N, \theta)$$

we can rewrite given set of equations:

$$Y_N = \bar{F}(U_N, \theta)$$



# Determination of the plant parameters (5)

Solution of  $Y_N = \bar{F}(U_N, \theta)$  gives us identification algorithms:

$$\theta = \bar{F}^{-1}(U_N, Y_N) \stackrel{df}{=} \Psi_N(U_N, Y_N)$$

where:

$\bar{F}^{-1}$  - inverse function

$\Psi_N$  - identification algorithm

$$N \times L \geq R$$



# Determination of the plant parameters (6)

## Example

Model

$$y = F(u, \theta) = \theta^T f(u)$$

where:

$$f(u) = \begin{bmatrix} f^{(1)}(u) \\ f^{(2)}(u) \\ \vdots \\ f^{(R)}(u) \end{bmatrix}$$

and condition:

$$N = R$$





# Determination of the plant parameters (7)

For given model system of equations has the form:

$$y_n = \theta^T f(u_n), \quad n = 1, 2, \dots, R$$

and can be rewritten:

$$Y_R^{df} = [y_1 \quad y_2 \quad \dots \quad y_R] = [\theta^T f(u_1) \quad \theta^T f(u_2) \quad \dots \quad \theta^T f(u_R)]$$

or

$$Y_R^T = \bar{f}^T(U_R)\theta$$



# Determination of the plant parameters (8)

where:

$$\bar{f}(U_R) \stackrel{df}{=} [f(u_1) \quad f(u_2) \quad \cdots \quad f(u_R)]$$

and identification condition:

$$\det[\bar{f}^T(U_R)] \neq 0$$

Identification algorithm has the form:

$$\theta = \Psi_R(U_R, Y_R) = [\bar{f}^T(U_R)]^{-1} Y_R^T$$



# Determination of the plant parameters (9)

Special case:  $f(u) = u$ , linear characteristic.

In this case system of equations has the form:

$$Y_R^T = U_R^T \theta$$

identification algorithm:

$$\theta = \Psi_N(U_R, Y_R) = [U_R^T]^{-1} Y_R^T$$

with condition:

$$\det[U_R] \neq 0$$



# Thank you for attention

